Giant number fluctuations in locally hyperuniform states

From flocking birds to migrating cells, Leiden 2025

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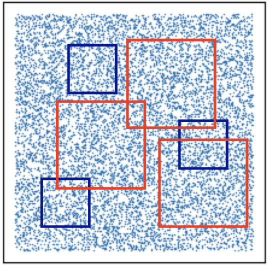
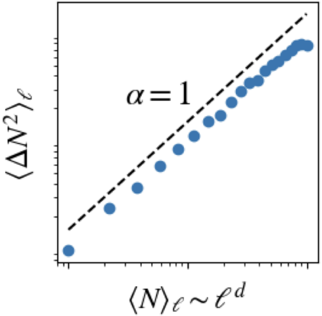
Number fluctuations

How are particles distributed in space?





- Short range correlations $\Rightarrow$ central limit theorem $\Rightarrow \langle \Delta N^2 \rangle_\ell \sim \langle N \rangle_\ell^\alpha$ with $\alpha = 1$
- Applies in equilibrium away from critical points

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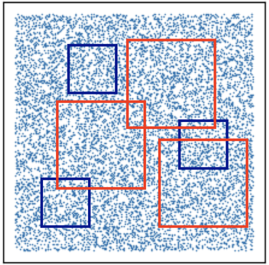
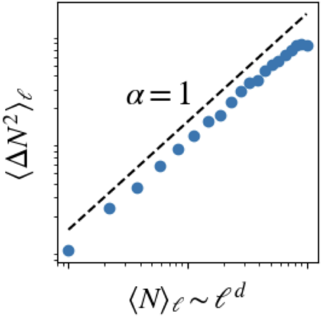
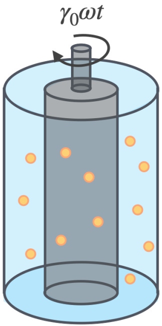




- Anomalous fluctuations: violation of the central limit theorem $\alpha \neq 1$
- Frequent out of equilibrium [*Derrida, JSM '07, Schmittmann, PRL '91, Grinstein, PRL, '90*]

An inspiring experiment

Periodically sheared suspension of large particles, Re $\ll$ 1. [*DJ Pine et al., Nature '05*]



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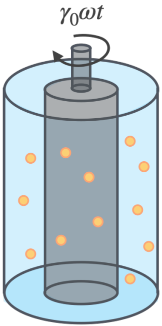
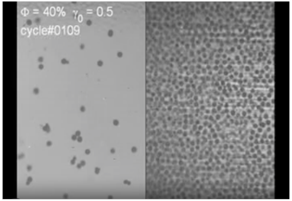


Low shear $\gamma_0 < \gamma_0^c$



Reversible dynamics
Stroboscopically arrested

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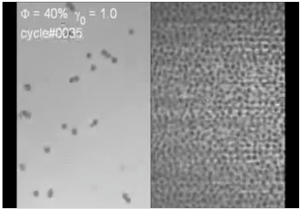

Irreversible dynamics
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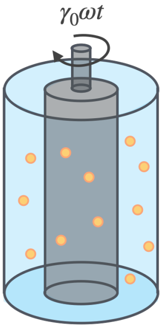

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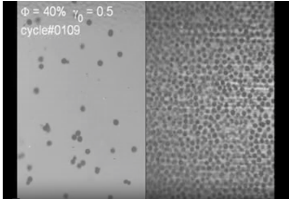

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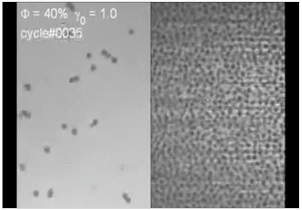

Irreversible dynamics
Stroboscopically active

- Absorbing phase transition
- At criticality, the system is hyperuniform i.e. $\langle \Delta N^2 \rangle \sim \langle N \rangle^\alpha$ with $\alpha < 1$.

Hyperuniformity: $\alpha < 1$

Suppression of number fluctuations at large length scales, $\langle \Delta N^2 \rangle \sim \langle N \rangle^{\alpha}$ with $\alpha < 1$.

Intuition: Self-organization to avoid collisions $\Rightarrow$ suppressed fluctuations

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Intuition: Self-organization to avoid collisions $\Rightarrow$ suppressed fluctuations

- Emergent in driven suspensions close to absorbing phase transition
- Trivial in crystals ($\alpha = 1/2$ in 2D)
- Engineered in disordered materials

  [Florescu et al., PNAS '09, Xie et al., PNAS '13, Shi et al., PRE '23]

Giant number fluctuations $\alpha > 1$

Amplification of fluctuations at large length scales, $\langle \Delta N^2 \rangle \sim \langle N \rangle^{\alpha}$ with $\alpha > 1$.

A universal property of orientationally-ordered active matter

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Amplification of fluctuations at large length scales, $\langle \Delta N^2 \rangle \sim \langle N \rangle^\alpha$ with $\alpha > 1$.

A universal property of orientationally-ordered active matter

- Predicted: mean-field linear theory for active nematics: $S(q) \underset{q \to 0}{\sim} q^{-2}$

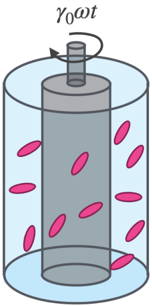
  [S. Ramaswamy et al., Europhys. Lett. '03]

- Measured: nematic rods, polar disks, epithelium tissue, bacterial colony...

  [Deseigne et al., Soft Matter '12, Zhang et al., PNAS '10, Giavazzi et al., J. Phys. D: Appl. Phys. '17]

Question: How does activity couple with absorbing phase transition?

- Theoretical: Interplay between giant number fluctuations and hyperuniformity?
- Experimental e.g. assemblies of biological cells under stress-induced motility, subcritical Quincke rollers, periodically sheared rod-like particles



[Franceschini et al., Soft Matter '14]

Active Nematic Random Organization Model (ANROM)

Random organization model $+$ alignment$+$ activity

Position $\boldsymbol{r}_i^t$, nematic director $\boldsymbol{n}_i^t = (\cos\theta_i^t, \sin\theta_i^t)$ with $\theta_i^t \in [-\pi/2, +\pi/2]$

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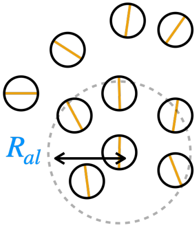

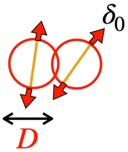

$$\theta_i^{t+1} = \frac{1}{2}\mathrm{Arg}\left[\sum_{k\in\mathcal{V}_i} e^{\mathrm{i}2\theta_k^t}\right] + \psi_i^t \qquad \text{(Vicsek-type)}$$

$$\boldsymbol{r}_i^{t+1} = \boldsymbol{r}_i^t \pm \delta_0 \boldsymbol{n}_i^t \qquad \text{if overlapping}$$

Noise $\psi_i^t \in [-\sigma\pi/2, \sigma\pi/2]$ and $0 \leq \sigma \leq 1$ noise strength.

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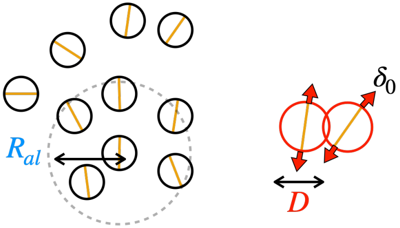

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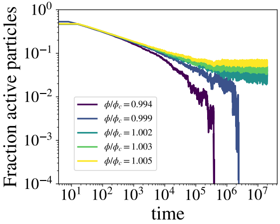
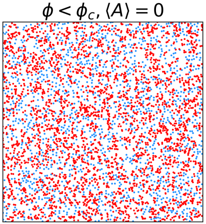
Known limits:

- For $\sigma \to 1 \Rightarrow$ Random organization model [*Tjhung et al., PRL '15*]
- For diameter $D \to \infty \Rightarrow$ Dry active nematics

Phenomenology

- Packing fraction $\phi < \phi_c \Rightarrow$ absorbing state with fraction of active particle $A = 0$





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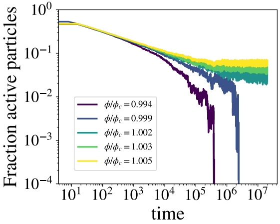

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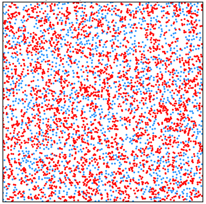


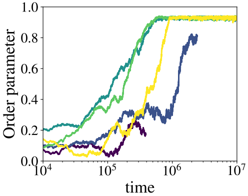
- Large densities, small angular noise $\Rightarrow$ nematic order

Nematic order parameter:

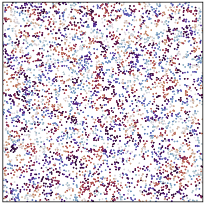
$S = 2\sqrt{|\det \boldsymbol{Q}|}$

Nematic tensor:

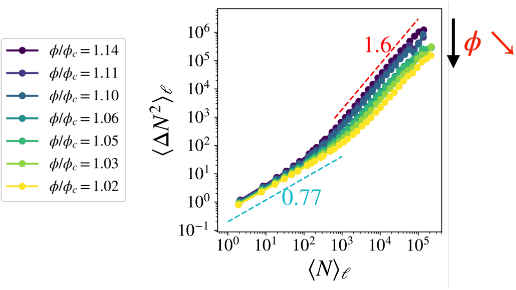
$\boldsymbol{Q} = \mathrm{N}^{-1} \sum_i \left(\boldsymbol{n}_i^t \otimes \boldsymbol{n}_i^t - \frac{1}{2}\mathbf{1} \right)$



Orientation



Two regimes of anomalous fluctuations



- Intermediate scales, $\alpha = 0.77 < 1 \rightarrow$ Hyperuniformity
- Large scales, $\alpha = 1.6 > 1 \rightarrow$ Giant number fluctuations

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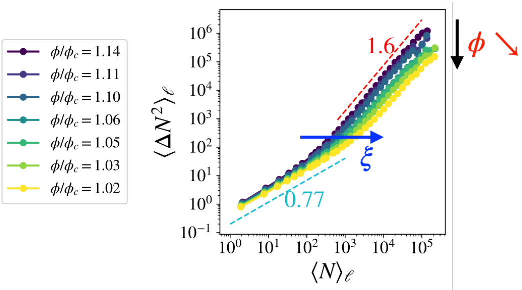

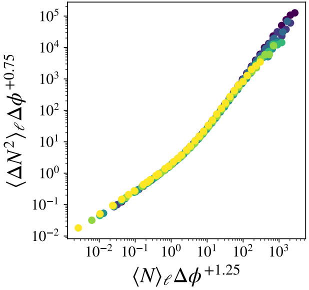

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- Growing cross-over length $\xi \underset{\phi \geq \phi_c}{\sim} (\phi - \phi_c)^{-\mu}$
- Collapse of the data: $\mu \approx 0.625$

Predicting the scaling regimes using a linearized fluctuating hydrodynamics

- Three fields: density of active particles, total density, nematic field
- Quantifying density correlation via the structure factor

$$S(\boldsymbol{q}) \propto \langle \hat{\rho}(\boldsymbol{q}, t)\hat{\rho}(-\boldsymbol{q}, t)\rangle \underset{q\to 0}{\sim} q^{\lambda} \text{ with } \boxed{\lambda = (1-\alpha)d}$$

1. $\alpha < 1 \Rightarrow \lambda > 0 \Rightarrow$ Vanishing $S(q)$
2. $\alpha > 1 \Rightarrow \lambda < 0 \Rightarrow$ Diverging $S(q)$

Critical scaling

Non-trivial scaling relation at large lengthscale, close to criticality:

$$\Rightarrow \boxed{\frac{S(q,\theta)}{\Delta\phi^{1/2}} \propto \frac{(1-\cos 4\theta)\lambda^2\chi_2^2\sigma_Q^2}{D_Q D_\rho \kappa(\kappa D_\rho + \lambda D_Q)}\frac{1}{\tilde{q}^2} + \frac{2D_\rho\sigma_A^2}{\kappa\lambda}\tilde{q}^2} \quad \text{with } \tilde{q} = q\Delta\phi^{-3/4}$$

- If $q \gg \Delta\phi^{3/4}$, $S(q) \sim q^\lambda$ with $\lambda = +2$ (HF)
- If $q \ll \Delta\phi^{3/4}$, $S(q) \sim q^\lambda$ with $\lambda = -2$ (GNF)

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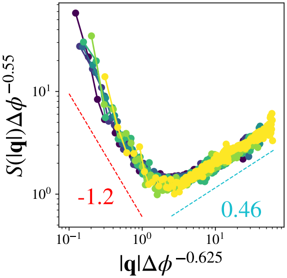


- Critical exponent: $\mu = \frac{3}{4}$ (predicted) versus $\mu = 0.625$ (measured)

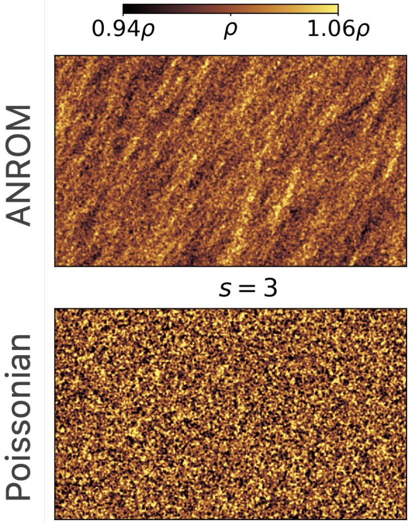
Noise competition

Phenomenological model:

$$\Rightarrow \quad \partial_t \delta\hat{\rho} = -Dq^2\delta\hat{\rho} + \underbrace{\frac{\tilde{\sigma}_A q^2}{\sqrt{A_0}}\hat{\eta}_A}_{\text{activity}} + \underbrace{\tilde{\sigma}_Q \sin 2\theta A_0 \hat{\eta}_Q^{\perp}}_{\text{nematic}}$$

- Activity field acts as superconservative noise with variance $\propto \frac{q^4}{\Delta\phi}$
- Nematic field acts as an (effective) non-conservative noise with variance $\Delta\phi^2$.
- Two noises comparable for $q^* \sim A_0^{3/4} \sim \Delta\phi^{3/4}$

Can *you see* the opposite regimes of anomalous fluctuations?



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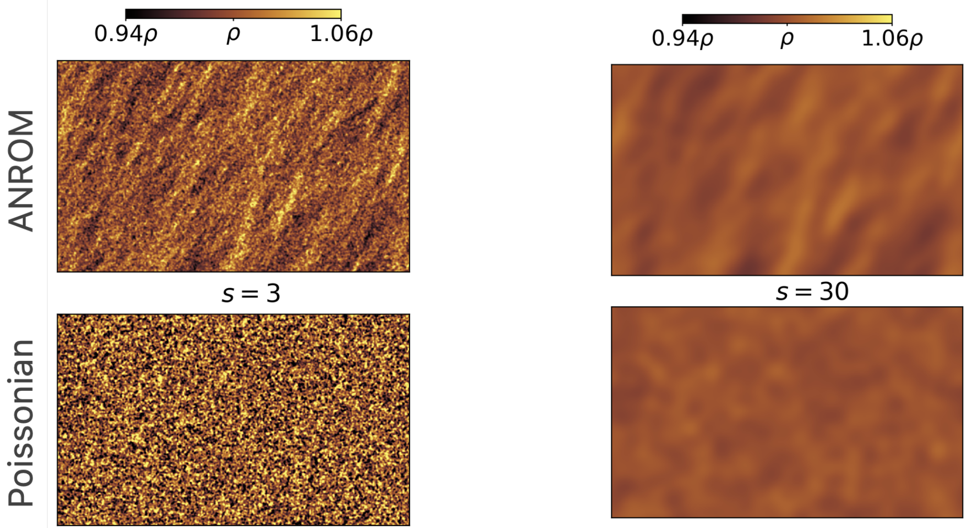

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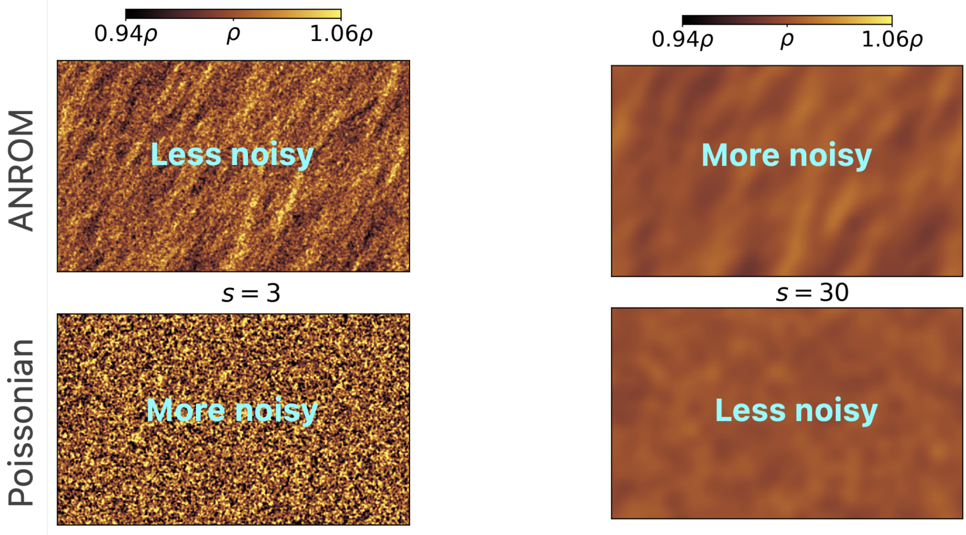

Conclusion... and thank you!

- Minimal active-matter model coupling nematic order and absorbing phase transitions
- Coexistence in Fourier space of opposite anomalous fluctuations
- General mechanism understood from linearized hydrodynamic theory

[Preprint arXiv:2410.18741]

The End